Variable Moment of Inertia Model for the CUSF 6DOF Trajectory Simulator

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# Overview

## Assumptions

* The vapour phase does not contribute to the moment of inertia.
* No sloshing occurs
* Fuel tank is a cylinder
* Liquids do not contribute to the moment of inertia of a rocket about its long axis (i.e. assume the liquids are inviscid, so they do not rotate with the rocket about the long axis).

## Summary of model

* Liquid fuel is represented by a cylinder, constant radius, with a height and density that changes with time.
* Solid fuel is modelled as an annular cylinder, constant density and outer radius, with an inner radius that increases with time.

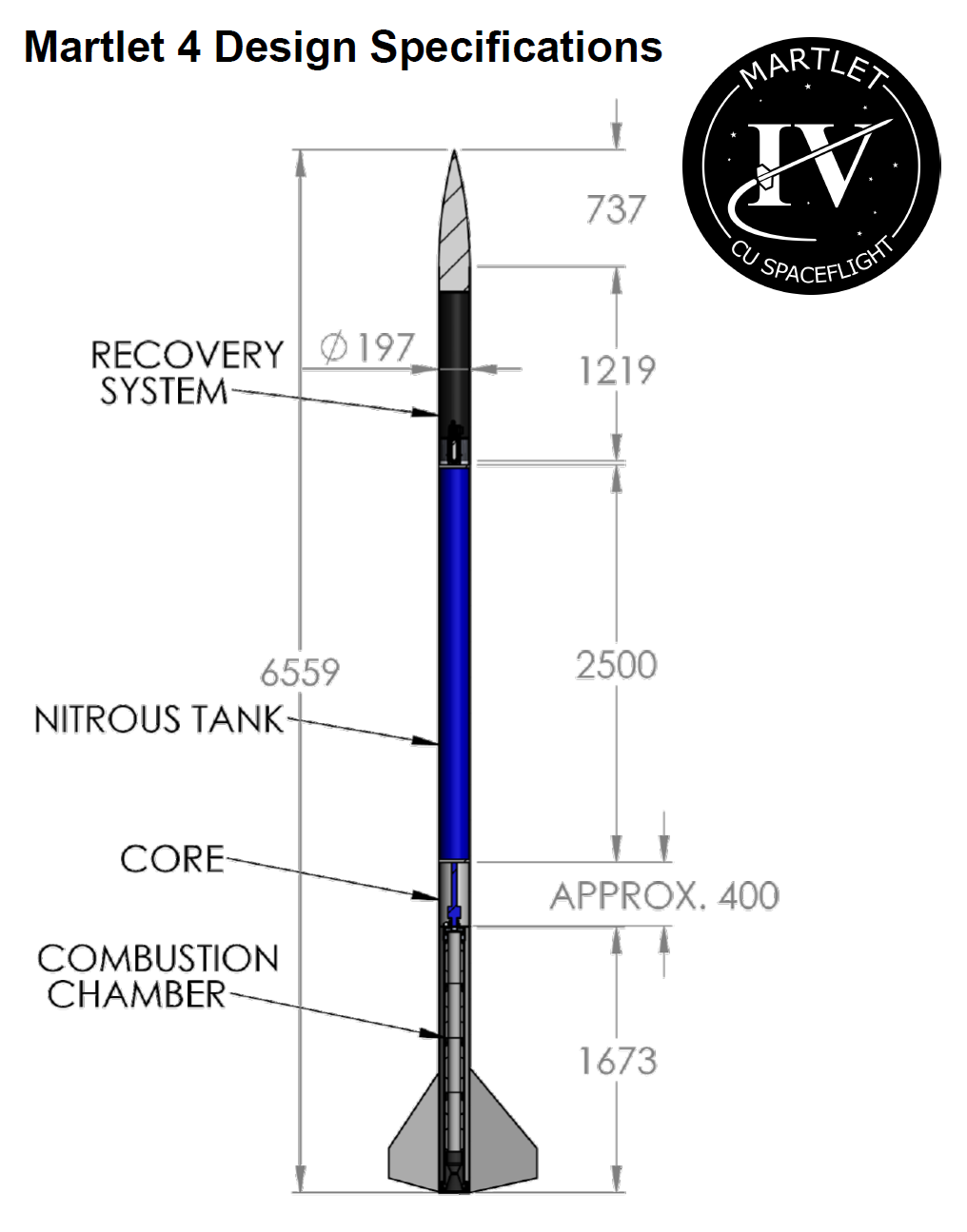


Figure 1: Martlet 4 geometry, from [1]

# Theory

## Moments of inertia for basic shapes

Equations obtained from the CUED Mechanics Databook [2], which also contains useful diagrams showing the geometry and definitions of symbols.

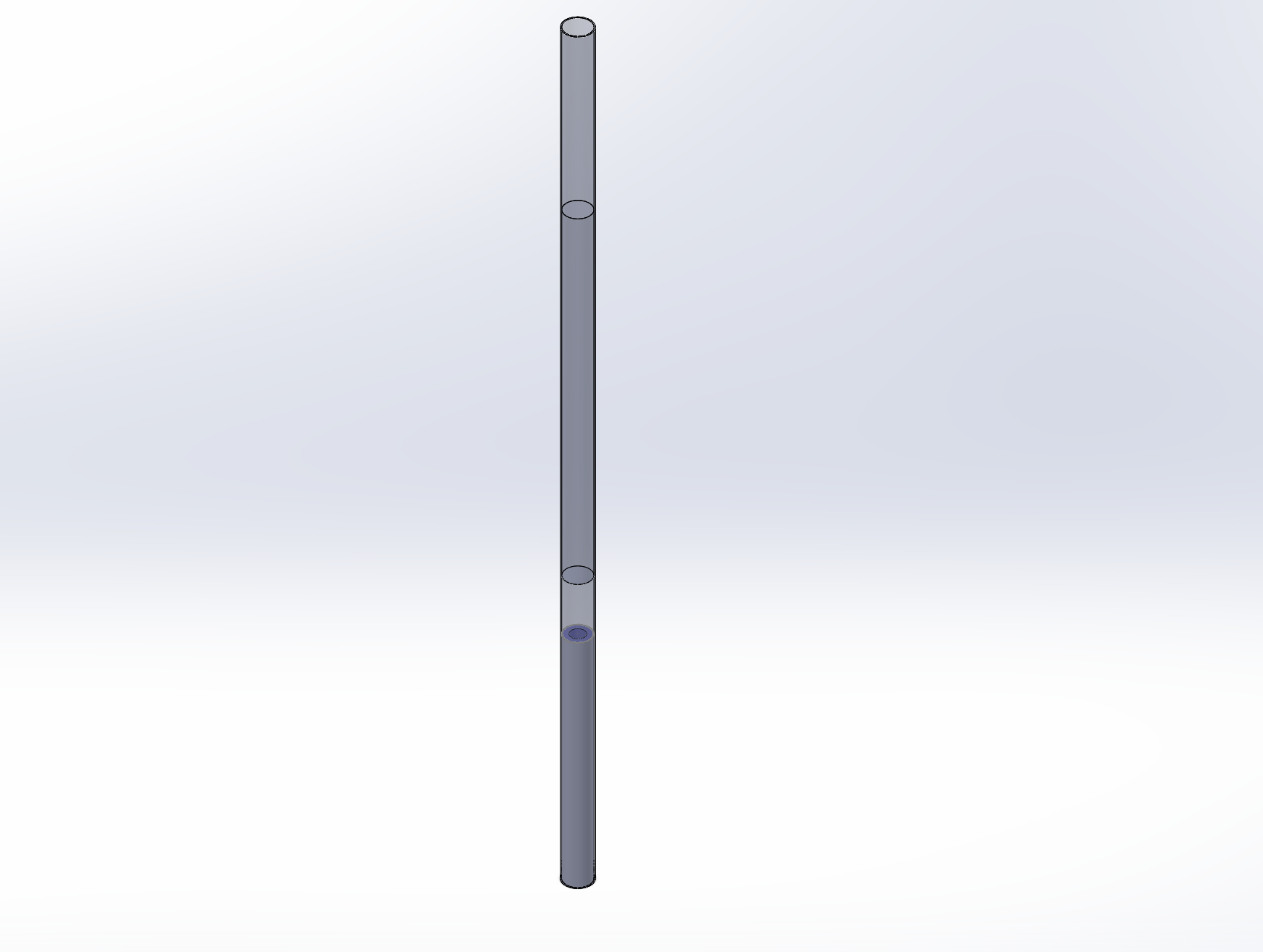
|  |  |  |
| --- | --- | --- |
| Shape | Moments of inertia | Definitions |
|  |  |  |
| Cylinder |  | radius |
| Annular cylinder |  | average radius  wall thickness  length |

## 

## Parallel axis theorem

Where are at the centre of mass.

## Geometry



Rocket shell

(thin-walled cylinder)

Liquid fuel

(solid cylinder)

Solid fuel

(hollow cylinder)

datum

Figure 2: Diagram of the model used

## Data available from Joe Hunt’s motor\_sim.py

The parameters output from Joe Hunt’s motor simulation gives us following data as a function of time:

* Liquid density, – referred to as *lden* in the code
* Liquid mass, – referred to as *lmass* in the code

Note however that, as far as I can tell, the script **does not currently include data on the mass of solid fuel**.

## Conventions

* Positions are measured relative to the bottom of the rocket in Cartesian .
* always refers to a distance from the bottom of the rocket.
* is used to denote the position of a centre of mass.
* is around the individual centre of mass for a component. is around the centre of mass of the whole rocket.

## Rocket Shell

The rocket shell (i.e. structural parts that don’t vary with time) will be assumed to have a constant moment of inertia. For this analysis, we will model it as a thin cylindrical shell. However, it would be fairly easy to get values for more complicated shapes, by creating a model in SolidWorks, and then using SolidWorks to calculate the principal moments of inertia.

The subscript ‘shell’ refers to ‘rocket shell’.

## Liquid Fuel

The subscript ‘l’ refers to ‘liquid’. Note that:

* We will assume the liquid is inviscid, meaning that **it will not rotate with the rocket** about its long axis – hence .
* The mass of the liquid cylinder, , is a function of time.
* The length of the liquid cylinder, , is a function of time.
* is the centre of mass at .
* is the length of the cylinder at .

Since it’s modelled as a solid cylinder:

|  |
| --- |
|  |

Geometry variation with time:

|  |
| --- |
|  |

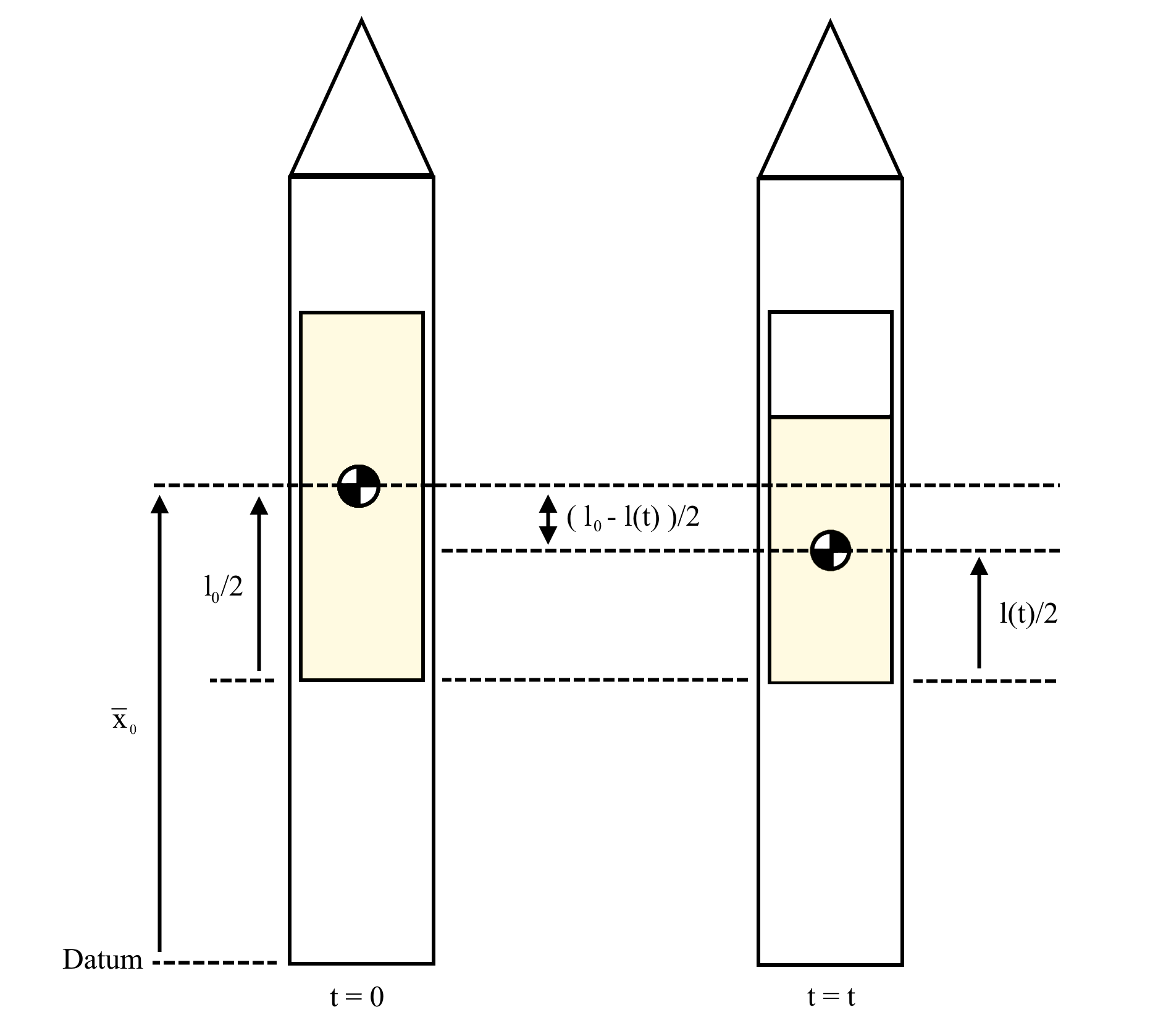


Figure : Definitions of lengths

|  |
| --- |
|  |

Knowing the position of the centre of mass, you can now reliably use the parralel axis theorem to find the moments of inertia about other points.

## Solid Fuel

# References

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| [1] | Cambridge University Spaceflight, Martlet 4 Specifications. |
| [2] | Cambridge University Engineering Department, Mechanics Databook, 2017. |